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EVALUATION OF SWIRL RATIO AND KARMAN NUMBER FOR VORTICES

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Results of calculations are shown pertaining to similarity numbers for vortex tubes. Experimental data pertaining to various prototype and model studies are investigated.

There are now several experimental test stands available for simulation of streams whirled about the vertical axis [1]. However, only a few studies have been made concerning the selection of similarity numbers for such streams. Here estimates for some characteristic cases of whirled jet flow induced by various types of vortex generators will be given.

Following an analysis of the Navier—Stokes equations of motion as basis, we introduce the dimensionless parameter S representing the vortex ratio [1]

$$S = \frac{\pi v r^2}{Q}. \tag{1}$$

In turbulent streams during rotation there appear secondary convection currents. Their effect on the principal motion cannot be disregarded here. Using the method of scale conversions, we reduce the Reynolds equations to the dimensionless form

$$\frac{\partial \mathbf{v}_0}{\partial t} + (\mathbf{v}_0 \nabla)\, \mathbf{v}_0 = -\frac{1}{\rho} \nabla P_0 + \nu \Delta \mathbf{v}_0 + \operatorname{div}(-\mathbf{v}'\mathbf{v}').$$

The scale for nondimensionalizing the fluctuation velocities we select according to Truesdell [2], viz.

$$\mathbf{v}' = \sqrt{\frac{\mu\varepsilon}{\rho}}\, \hat{\mathbf{v}}'. \tag{2}$$

The variables we replace with normalized quantities $\hat{t}$, $\hat{\mathbf{r}}$, $\hat{\mathbf{v}}$, $\hat{P}_0$ as follows:

$$t = \frac{r_m}{v_m}\hat{t};\ \mathbf{r} = r_m \hat{\mathbf{r}};\ \mathbf{v} = v_m \hat{\mathbf{v}};\ P_0 = P\hat{P}_0;\ \rho = \rho_0 \hat{\rho};\ P = \rho_0 v_m^2.$$

It has been proposed [2] that additional viscous stresses in a stream are proportional to the ratio of viscous friction forces to normal pressure forces. The selected scale (2) yields an expression for characteristic turbulent stresses in terms of a relation between these forces. Omitting the symbol of dimensionlessness in the notation, we obtain

$$\frac{\partial \mathbf{v}_0}{\partial t} + (\mathbf{v}_0 \nabla)\, \mathbf{v}_0 = -\frac{1}{\rho} \nabla P_0 + \frac{1}{\mathrm{Re}} \Delta \mathbf{v}_0 + K \operatorname{div}(\overline{-\mathbf{v}'\mathbf{v}'}), \tag{3}$$

where $\mathrm{Re} = r_m v_m/\nu$ and $K = \mu\varepsilon/P$.

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In order to calculate the parameter K for specific cases, it is necessary to express eddy stresses through averaged flow parameters. A conversion from fluctuations to average velocities has been postulated [3] which takes into account hydrodynamic nonuniformity within a molar volume of averaging. The equations of motion for an incompressible fluid with the Coriolis force disregarded are

$$\frac{\partial \mathbf{v}_0}{\partial t}+(\mathbf{v}_0\nabla)\,\mathbf{v}_0=-\frac{1}{\rho}\,\nabla P_0+\frac{1}{\mathrm{Re}}\Delta\mathbf{v}_0+\beta\,(\mathbf{v}_0\nabla)\,\mathbf{v}_0. \tag{4}$$

Based on the concept of gyrotropism or mutual whirling of average and eddy motion, an equation [4] which is analogous to Eq. (4) has been derived. Together with the Reynolds equation for average motion, the Reynolds equation for eddy motion [5] was also considered.

From among all possible orientations of vectors $\mathbf{v}_0$ and $\mathbf{v}'$ in turbulent streams, we select those which correspond to mutual whirling of stream lines of eddy and average motion

$$(\mathbf{v}'\,\mathrm{curl}\,\mathbf{v}_0)\neq 0;\ (\mathbf{v}_0\,\mathrm{curl}\,\mathbf{v}')\neq 0. \tag{5}$$

Specifically, when a likely parallel orientation of vectors $\omega_0=\frac{1}{2}$ curl $\mathbf{v}_0$ and $\mathbf{v}'$ as well as of vectors $\omega'=\frac{1}{2}$ curl $\mathbf{v}'$ and $\mathbf{v}_0$ can be assumed, we obtain the relation

$$[\omega'\times\mathbf{v}']=-a(t)\,[\omega_0\times\mathbf{v}_0];\ [\omega'\times\mathbf{v}_0]=0;\ [\mathbf{v}'\times\omega_0]=0. \tag{6}$$

Insertion of expressions (6) into the Reynolds equation for an incompressible fluid yields an equation analogous to Eq. (4).

We define the Predvoditelev parameter as

$$\beta=\alpha K. \tag{7}$$

Here α is a numerical constant equal to the average value of $\overline{a(t)}$.

Let us now compare the right-hand sides of the equations of motion where tangential stresses are expressed through the eddy viscosity ν_0 and the parameter β. We equate the right-hand sides of Eq. (4) for the velocity component v_φ and of the equations of motion with the viscosity coefficient [6]

$$\beta v_r\frac{\partial v_\varphi}{\partial r}+\beta v_r\frac{v_\varphi}{r}=\frac{1}{\mathrm{Re}_0}\left(\frac{\partial^2 v_\varphi}{\partial r^2}+\frac{1}{r}\,\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r^2}\right). \tag{8}$$

It has been assumed here that v_φ is not a function of the z-coördinate, the symbol of dimensionlessness has been omitted, and $\mathrm{Re}_0 = v_m r_m/\nu_0$.

The equation becomes an identity when $v_\varphi = 2\nu_0/\beta r$. Specifically for $r = r_\infty$ and $v = v_\infty$ we have

$$\nu_0=\frac{\beta}{2}\,v_\infty r_\infty. \tag{9}$$

As an example we consider here only one of the exact solutions to the Navier–Stokes equation, viz. the solution which describes a potential vortex.

According to the semiempirical theory of turbulence, we can write the following equation in a cylindrical system of coordinates [6]

$$\nu_0=-l^2\left(\frac{dv_\varphi}{dr}-\frac{v_\varphi}{r}\right);\ l=\varkappa\,\frac{\left(\frac{dv_\varphi}{dr}-\frac{v_\varphi}{r}\right)}{\frac{d}{dr}\left(\frac{dv_\varphi}{dr}-\frac{v_\varphi}{r}\right)}.$$

The tangential velocity at far distances from the center of a potential vortex is $v_\varphi=v_\infty r_\infty/r$. After inserting this expression into that for eddy viscosity, we have

$$\nu_0=\frac{\varkappa^2}{2}\,v_\infty r_\infty. \tag{9'}$$

Comparing expressions (8), (9), and (9'), we obtain

$$\beta=\varkappa^2\ \text{and}\ \varkappa^2=\frac{\alpha\mu\varepsilon}{P}. \tag{10}$$

TABLE 1. Comparative Vortex Characteristics

Type of vortex	v_m, m/sec	r_m, m	r_0, m	h, m	Γ_m, m²/sec	Γ_∞, m²/sec	Q, m³/sec	$\varkappa$	S_m	S	$r_0/2h$	Θ
Typhoon [11]	70	15000	$5 \cdot 10^5$	$3{,}2 \cdot 10^3$	$6{,}6 \cdot 10^6$	$12{,}6 \cdot 10^6$	$12{,}2 \cdot 10^{10}$	0,002	0,40	0,77	78	18
Tornado [12]	68	67	444	36	$2{,}8 \cdot 10^4$	$4 \cdot 10^4$	$3 \cdot 10^6$	0,012	0,32	0,45	6,2	25,5
Waterspout [13]	25	12	280	14	1884	2638	$4{,}2 \cdot 10^4$	0,031	0,27	0,38	19,5	24,1
Dust devil [14]	10	10	300	10,0	628	879	$8{,}4 \cdot 10^3$	0,04	0,38	0,52	15	46,3
Lab. vortices [9]												
test no. 1	1,84	0,094	0,61	0,438	1,09	1,52	1,4	0,37	0,04	0,33	0,7	25,5
test no. 2	2,05	0,196	0,61	0,438	2,52	3,53	1,4	0,28	0,18	0,7	0,7	47,7
test no. 1[15]	20,1	0,084	0,103	0,166	10,6	14,8	0,05	0,37	9,3	13	0,31	89
test no. 2	19,8	0,076	0,103	0,153	9,5	13,2	0,45	0,39	0,8	1,12	0,34	78
Mea. by these authors												
test no. 1	14,9	0,04	0,11	0,094	3,8	5,3	0,114	0,24	0,66	0,92	0,61	76
test no. 2	11,2	0,04	0,10	0,10	2,8	4,0	0,134	0,27	0,42	0,6	0,55	71

It is to be noted that all these considerations are valid when ν_0 remains constant not only at the vortex periphery but also in other vortex regions. This is evidently justified, inasmuch as in most cases turbulence is transferred from peripheral regions to the central region by radial flow. On the basis of the assumptions made here, therefore, there exists a proportionality between β (Predvoditelev number), $\varkappa$ (Karman number), and K (Truedell number).

We now transform relations (10) to equivalent ones containing quantities that can be measured in a vortex. From the equations of a boundary layer we obtain for the thickness δ of a vortical boundary layer [7]

$$\delta \simeq 8\left(\frac{\nu_0}{\Omega}\right)^{1/2}.$$

Let $\Omega = v_m/r_m$. Then $\delta \simeq 6.69\varkappa r_m$. In expressions (10) we let $P = \rho v_m^2/2$ and $\varepsilon = v_m/\delta$. Using the numerical constant $\alpha = 1.9 \cdot 10^3$ [8], we obtain

$$\varkappa = 8.3\left(\frac{\nu}{v_m r_m}\right)^{1/3}. \tag{10'}$$

The quantity S for vortices can be expressed through circulation Γ, flow rate Q, and the characteristic cross-sectional dimension of a vortex [1]

$$S = \frac{\Gamma \cdot r}{2Q}.$$

The quantity S can be calculated in several different ways. The circulation can be defined within the region of potential flow at $r = r_\infty$, viz. $\Gamma_\infty = 2\pi r_\infty v_\infty$. The cross-sectional dimension is identified with the radius r_0 of the convection region. Now [1]

$$S = \frac{\Gamma_\infty r_0}{2Q} = \frac{r_0}{2h}\,\mathrm{tg}\,\theta, \tag{11}$$

where $\tan\theta = v_\infty/u_\infty$.

The lack of reliable data on Γ_∞ and r_0 is responsible for a wide spread of S values. It will be more exact to calculate S from the circulation Γ_∞ at the boundary of the vortex core, viz. at $r = r_m$:

$$S_m = \frac{\Gamma_m r_m}{2Q}. \tag{12}$$

Results of calculations of S and S_m according to expressions (11) and (12) are given, for comparison, in Table 1. Included here are also values of $\varkappa$ according to relation (10').

The swirl ratio S_m for natural vortices of various types fluctuates very little, on the average, about the value 0.34 approximately. The test stand at the Institute of Heat and Mass Transfer (Academy of Sciences of the Belorussian SSR), which the author used for measurements in test 2, had, according to the data in Table 1, the capability for reaching S_m values nearest to those encountered in nature. An artificial deepening of the inrush zone as in study [9], for instance, has little effect on the actually attainable values of S_m.

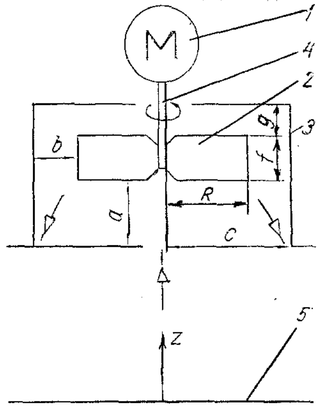

Fig. 1. Schematic diagram of vortex generator: 1) electric motor; 2) four-blade turbulizer; 3) cowl; 4) motor shaft; 5) bottom surface. Parameters of vortex generators: Moscow State University (test 1) a = 0.03 m, b = 0.024 m, c = 0.105 m, f = 0.07 m, g = 0.035 m, R = 0.081 m; Institute of Heat and Mass Transfer AS BSSR (test 2) a = 0.03 m, b = 0.0305 m, c = 0.105 m, f = 0.0695 m, g = 0.022 m, R = 0.059 m.

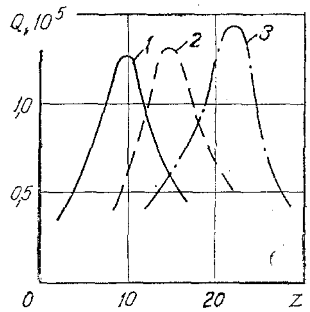

Fig. 2

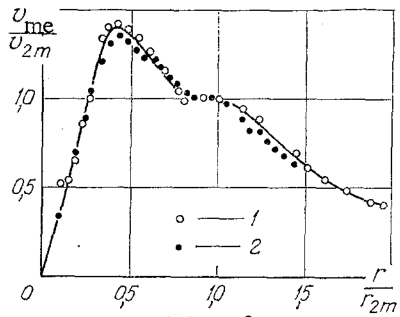

Fig. 3

Fig. 2. Curves of total flow rate (volume per unit time) $Q\cdot 10^5$ cm^3/sec through vortex tube along axis z, cm: 1) l = 23 cm, ω = 6000; 2) l = 29, ω = 3200 rpm; 3) l = 36 cm, ω = 3200 rpm.

Fig. 3. Radial profiles of tangential velocity: 1) data in [9] (test 2), 2) measurements by these authors (test 1).

This has evidently to do with the dependence of the flow structure in laboratory vortices on the Karman number $\varkappa$. When the latter approaches unity, then these vortices acquire a unique hydrodynamic structure [10]. The tangential velocity $v_\varphi(r)$ has here two maxima. The pressure profile $\Delta P(r)$ contains a spike corresponding to a high-pressure zone at the periphery. The axial component of the velocity curl $\omega_z(r)$ reverses sign.

Let us compare the $v_\varphi(r)$ profiles in vortices of this type according to measurements in study [9] and measurements by these authors. The characteristic parameters of laboratory apparatus are given in Fig. 1. It is to be noted that tests 2 were performed in a stand with clearance between motor shaft and cowl lid. Tests 1 were performed with a vortex generator containing a 1.5-cm-wide annular gap. The presence of this gap affected the resultant rate of air flow through the vortex tube. The graph in Fig. 2 depicts the variation of flow rate Q(z) equal to the algebraic sum of volumes per unit time Q_1 and Q_2 ascending and descending, respectively.

At certain distances z = h the flow rate Q(z) reached its maximum value. These distances z = h corresponded, evidently, to the height to which air systematically entered the vortex tube. The graph in Fig. 3 depicts, in dimensionless coordinates, the profiles of tangential velocity in various test stands. It is evident here that the profiles are the same, although the swirl ratios differ.

The Karman number for the cases compared here is equal to 0.25 approximately. A change of its value would result in increasing the difference between the $v_\varphi(r)$ profiles. The results obtained here indicate that in order to establish a correspondence between vortices of different physical nature, it is necessary to achieve their dynamic similarity not only with respect to parameter S but also with respect to the Karman number $\varkappa$.

NOTATION

S, swirl ratio; $Q = 2\pi \int_0^{r_0} v_z r dr$, flow rate (per second) through a vortex tube; v_φ, v_r, v_z, respectively, the tangential component, the radial component, and the axial component of velocity; v_0 and P_0, velocity and the pressure of average flow; v', velocity of eddy flow; μ, dynamic viscosity; ρ, density; ν/μρ, kinematic viscosity; ν_0, eddy viscosity; $r = r_m$, radius at the first maximum of tangential velocity; L, length of a vortex; δ, thickness of the boundary layer; h, height of stream entrance into a vortex; ε, velocity gradient; Re, Reynolds number; K = με/P, Truesdell number; β, Predvoditelev number; $\varkappa$, Karman number; α, a numerical constant; l, mixing length; v_∞, tangential velocity at $r = r_\infty$; $\Omega = v_m/r_m$, angular velocity in a vortex at $r = r_m$; v_m, tangential velocity at $r = r_m$; v_{2m}, tangential velocity at its second maximum, at $r = r_{2m}$; $\Gamma_\infty = 2\pi v_\infty r_\infty$, circulation within the region of potential flow; $\Gamma_\infty = 2\pi v_m r_m$, circulation at the boundary of the vortex core, at $r = r_m$; and $\tan\Theta = v_\infty/u_\infty$, angle of stream entrance into a vortex tube within the region of potential flow.

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